Homogenization of interlocking masonry walls

Ioannis Stefanou, Jean Sulem, Ioannis Vardoulakis

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Ioannis Stefanou (istefanou@mechan.ntua.gr)
Department of Applied Mathematics and Physics, National Technical University of Athens, Athens, Greece

Jean Sulem (sulem@cermes.enpc.fr)
UR Navier-CERMES, Ecole Nationale des Ponts et Chaussées, Paris, France

Ioannis Vardoulakis (I.Vardoulakis@mechan.ntua.gr)
Department of Applied Mathematics and Physics, National Technical University of Athens, Athens, Greece

ABSTRACT. In a previous publication the authors proposed a continuous model for describing the mechanical behaviour of 'Running Bond' masonry walls. Here a different masonry structure is considered. This structure is 'bi-atomic' in the sense that its pattern consists in two kind of blocks with different. The homogenization by differential expansions technique is used for obtaining an equivalent Cosserat continuum. It is shown that the enriched kinematics of the Cosserat continuum is not sufficient to capture the full dynamic behaviour of this bi-atomic system. However, the Cosserat continuum model behaves well for low frequency waves and for wave lengths 5 to 10 times bigger the elementary cell.

1. Introduction

Masonry is a two-phase composite material formed by regularly distributed bricks and mortar following a building pattern. In a previous study, the authors (Stefanou et al. 2007) have derived the constitutive law of a 3D-Cosserat continuum that describes both the in-plane and the out-of-plane deformation of a 'Running Bond' masonry wall pattern (Figure 1) and have examined the full dynamic behaviour.

Here a different pattern is investigated (Figure 1). The particularity of this pattern is that it is constituted by two different in size interacting blocks. This case is more general than the case of the 'Running Bond', which can be seen as degenerated case of the presented general pattern. However, the main characteristic of this pattern is the interlocking of the building blocks. Obviously, the absence of continuous joints increases the in-plane resistance of the masonry wall (Figure 2).

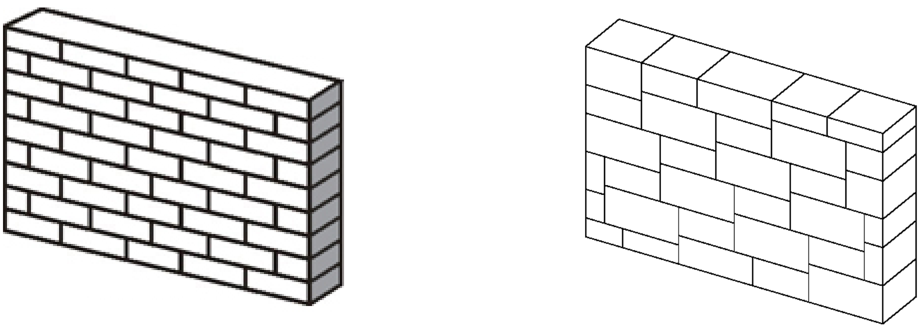

Figure 1. On the left a running bond masonry wall and on the right the interlocking masonry wall presented herein.

The homogenization procedure followed here results in an anisotropic Cosserat continuum. The enriched kinematics of the Cosserat continuum allows to model microelement systems undergoing in-plane rotations. The derivation of the constitutive law is based on the construction of a continuum, which, for any 'virtual' translational- and rotational- field, stores the same elastic energy as the corresponding discrete structure. The domain of validity of the resulting continuum is discussed by comparing the dispersion function of the discrete system of blocks with the continuous one.

Figure 2. Interlocking blocks at masonry wall in Peru. (Photo: I.Vardoulakis 2006)

2. The discrete model

The studied masonry wall is constituted by two types of blocks. The first one, i.e. the 'small block', has dimensions a_1 x b_1 x d, while the second, i.e. the 'big block', has a_2 x b_2 x d, where d is the thickness of the wall. Without any loss of generality, it holds: $a_2 \geq a_1$ and $b_2 \geq b_1$ (Figure 3).

Each block is considered rigid with deformable interfaces (soft-contacts). Therefore the deformation is concentrated at the interfaces of the blocks is small as compared to their dimensions. Raffard (2000) has experimentally shown that the rigidity of the interface (brick-mortar-brick) is smaller than the rigidity of the mortar itself. According to Raffard this may be attributed to an increased porosity at the interface mortar-brick. The developed stresses at the interfaces of the blocks are assumed to be linearly distributed over them and the constitutive law of the joints is assumed to be linear elastic. The assumption of linear stress distribution is justified in the recent publication of Milani et al. (2006), where the authors show that linear stress distributions at the interfaces give good results as compared to constant and quadratic stress distributions.

The arrangement of the building blocks is periodic in space and it follows a given pattern which connects their centres (nodes). In solid state physics terms (Kittel 1995), this pattern is called "lattice" while the repeated cell is called "basis":

$$lattice + basis = structure \tag{14}$$

The basis or the "elementary cell", is defined as the recurrent volume of the structure that contains all the necessary information for its constitutive description. It has to be mentioned though, that generally the elementary cell is not unique and that its choice affects the obtained homogenized continuum. For this rather well known point we refer to the book of Novozhilov (1961).

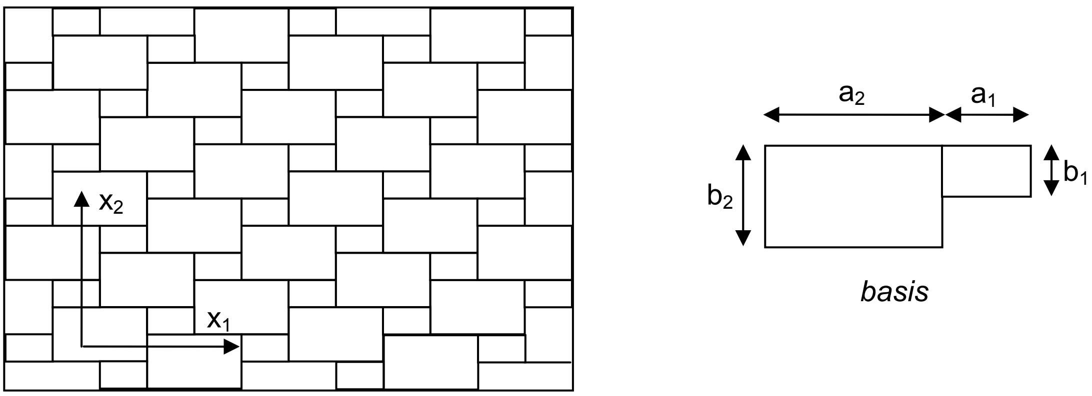

Figure 3. On the left the interlocking mosonry structure and on the right its basis.

In two dimensions (in-plane deformation), each block '*p*' of the elementary cell has two translational ($v_1^{(p)}, v_2^{(p)}$) and one rotational ($\phi_3^{(p)}$) degree of freedom. Respectively, $v_i^{(p)}$ and $\phi_3^{(p)}$ represent the displacement of the centre mass and the rotation of the block '*p*'. The displacement $v_i^{(p)}(r_j)$ of a point $r_j^{(p)}$ of the rigid block '*p*' is given for infinitesimal rotations by the following relation:

$$v_i^{(p)}(r_k^{(p)}) = v_i^{(p)} + e_{ijk}\phi_3^{(p)}\left(r_k^{(p)} - c_k^{(p)}\right) \tag{15}$$

where e_{ijk} is the Levi-Civita tensor, $c_k^{(p)}$ the centre of mass of the block '*p*' and $v_i^{(p)}, r_j^{(p)}, c_j^{(p)}$ are expressed in the global coordinate system.

Assuming linear stress distributions at the interfaces of the structure, we can substitute the stresses with punctual forces and moments at the centre of area of each interface. Let b^A, b^B be the two blocks interacting through interface $\Sigma^{(k)}$ and $F_i^{b^A,k}$ (resp. $F_i^{b^B,k}$) and $M_i^{b^A,k}$ (resp. $M_i^{b^B,k}$) the force and the moment exerted by block b^B over b^A (resp. b^A over b^B). The elementary cell interacts with the adjacent cells with ten 'external' interfaces $\Sigma^{(1)}$-$\Sigma^{(10)}$ while the two blocks of the basis interact through interface $\Sigma^{(0)}$. This set of interaction forces and moments is self-balanced and is expressed as follows:

$$\begin{aligned} F_i^{p,k} &= \mathcal{K}_{ij}^k \, \Delta v_j^k \\ M_i^{p,k} &= C_{ij}^k \, \Delta \phi_j^k \end{aligned} \tag{16}$$

where Δv_j^k and $\Delta \phi_j^k$ are respectively the relative displacement and relative rotation of the blocks at the centre of the area of interface *k* and $\mathcal{K}_{ij}^k$, C_{ij}^k are respectively the stiffness and the bending stiffness of the interface *k*.

Using D'Alembert's principle we can derive the equations of motion for each individual block of the masonry structure.

3. The Cosserat model

The additional rotational degrees of freedom of the Cosserat continuum make it suitable for describing materials with internal structure. The homogenization procedure followed here is based on the construction of a Cosserat continuum, which, for any 'virtual' translational- and rotational- field, stores the same elastic energy as the corresponding lattice structure.

The elementary cell considered for the Cosserat homogenisation is shown at Figure 4.

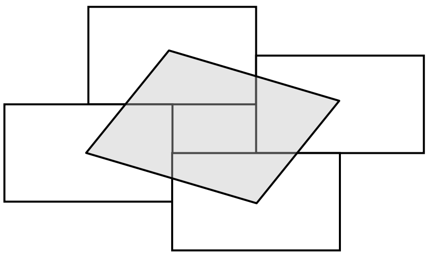

Figure 4. Elementary cell considered for the homogenisation with Cosserat continuum.

Let $\mathbf{u}^{cf} = \mathbf{u}^{cf}\left(x_1, x_2\right)$ and $\boldsymbol{\omega}^{cf} = \boldsymbol{\omega}^{cf}\left(x_1, x_2\right)$ be C^2 translational and rotational fields, such that their values are identical to those of the displacements and rotations at the centre mass of the small block of the lattice model. Expanding the continuous fields $\hat{\mathbf{u}}^{cf}$ and $\boldsymbol{\omega}^{cf}$ in Taylor series around $\mathbf{c}^{(p)}$ and up to the 2nd and 1st order respectively it is possible to substitute the discrete quantities with continuous ones that correspond to the deformation measures of the Cosserat continuum. Then the derivation of the constitutive law is straightforward. For more details in the homogenisation method used here the reader is referred to Stefanou et. al. (2007).

4. Dispersion functions

The dynamic response of a structure is characterized by the dispersion functions that relate the wave propagation frequency to the wavelength. For linear elastic behaviour it is possible to derive analytically the dispersion function of the lattice- and of the continuous systems by using respectively discrete and continuous Fourier transforms. If $\hat{\omega}$ is the normalized wave frequency and k the wave number of a wave travelling in the x_1 direction, the dispersion functions $\hat{\omega} = \hat{\omega}(k)$ are presented on Figure 5. Similar results are obtained for waves travelling in other directions. In the numerical example, presented below, the dimensions of the blocks are a_1 = 195mm, b_1 = 95mm, d = 190mm, a_2 = $2a_1$, b_2 = $2b_1$ and their specific weight is 20 kN/m^3. The thickness of the joints is taken 10mm and the Young Modulus of the mortar is 4GPa and its Poisson's ratio 0.2.

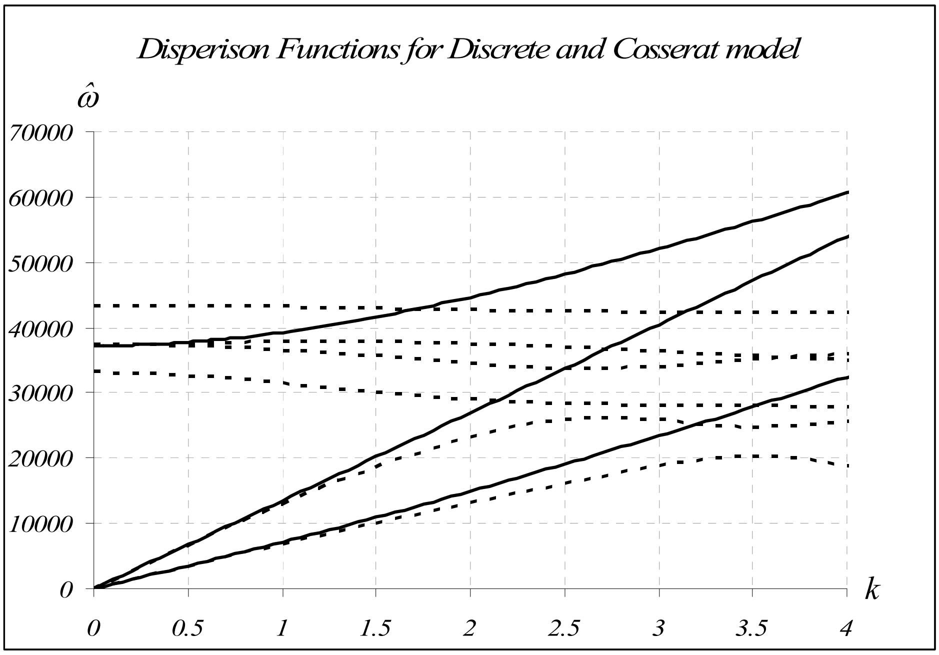

Figure 5. Dispersion curves for the Discrete (dotted lines) and Cosserat model (solid lines). Each curve represent a different oscillation mode.

Each one of the dispersion curves corresponds to a different oscillation mode, which combines rotation and translation of the two blocks of the 'bi-atomic', interlocking masonry structure. Notice, that the Cosserat model has only three oscillation modes, while the discrete structure has six. Thus the Cosserat kinematics is not sufficient to represent all the oscillation modes. However, the homogenised continuous model behaves well for low frequency waves and for relatively large wave lengths (small wave numbers).

5. Conclusions

In this paper a discrete and a continuous model of an interlocking masonry structure are presented. The aim was to start from the micro-scale, i.e. the blocks, and to derive a Cosserat continuum with equivalent mechanical characteristics. This was achieved and the domain of validity of the derived Cosserat continuum was examined in terms of the dispersion functions of the models. It was found that the Cosserat model behaves well for low frequency waves and for wave lengths 5 to 10 times bigger the elementary cell. In order to capture all the oscillation modes of the structure, higher order continua with microstructure have to be considered, but this exceeds the limits of this short paper.

6. References

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